Oscillating universes as eigensolutions of cosmological Schrödinger equation

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Abstract

We propose a cosmological model which could explain, in a very natural way, the apparently periodic structures of the universe, as revealed in a series of recent observations. Our point of view is to reduce the cosmological Friedman–Einstein dynamical system to a sort of Schrödinger equation whose bound eigensolutions are oscillating functions. Taking into account the cosmological expansion, the large scale periodic structure could be easily recovered considering the amplitudes and the correlation lengths of the galaxy clusters.

Key–words: cosmology, quantum mechanics, large scale structure, general relativity, Schrödinger equation.

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1   Introduction

Understanding the large scale structure of the universe is one of the main issue of modern cosmology. In fact, a large amount of observations tell us that the *cosmological principle* breaks at scales of the order $\sim 100h^{-1}$Mpc and below, while, well above this limit, ($\sim 1000h^{-1}$Mpc) it seems that the matter in the universe is homogeneously and isotropically distributed (Kolb & Turner 1990). The symbol h indicates the normalized Hubble constant whose values are $0.4 \leq h \leq 1$.

From a cosmological point of view (*i.e.* we are considering scales where galaxies are pointlike constituents), the clustering properties of objects clearly show that a sort of hierarchy exists: we have tight and loose groups of galaxies, clusters, superclusters and filaments of galaxies. All these structures come from an excess of correlation over given distributions and they can be well represented by the two–point correlation function

$$\xi_j(r) = C_j r^{-\mu_j} \, , \tag{1}$$

which fixes the correlation scales (Kolb & Turner 1990). The index j can be $j = G, C, S$ if the correlation is between galaxies, clusters, or superclusters. C_j determines the correlation amplitude which can be from 0.1 to $10h^{-1}$Mpc for galaxies, up to $25h^{-1}$Mpc for clusters, and up to $100h^{-1}$Mpc for superclusters (Peebles 1993). The exponent μ_j assignes the slope of the power law and it is found that, for all the hierarchical orders, it is the same, that is $\mu_j \simeq 1.8$. In other words, the excess of correlation is the "same" for galaxies, clusters and superclusters. This fact could mean that the distribution of objects is absolutely non–random and that some fundamental mechanism has led to the formation of large–scale structure.

A further evidence of such a point of view is the result by Broadhurst *et al.* (from now on BEKS) (Broadhurst *et al.*1990) which found an excess of correlation and an apparent regularity in the galaxy distribution with a characteristic scale of $128h^{-1}$Mpc. The structure was revealed after the completion of deep pencil beam surveys extending to red–shift $z > 0.2$ toward northern and southern Galactic poles (Broadhurst *et al.*1990). The linear extension of the surveys was to $2000h^{-1}$Mpc so that, the scale of $128h^{-1}$Mpc between successive density peaks could be interpreted as a sort of transition scale to homogeneity and isotropy at larger scales.

Several interpretations have been given to this seminal result. For example, cosmological solutions with an overall Friedmanian expanding behaviour, corrected by a small oscillatory regime, could implement such a structure (Busarello *et al.*1994). Furthermore, dynamics of a nonconformal scalar field could modify cosmological evolution in the sense of oscillations (Morikawa 1991), (Capozziello *et al.*1996).

It emerges that several observational quantities can be affected by such a behaviour since oscillations in the cosmological solutions can be easily reduced to oscillations in the red–shift z. In fact, if $a(t)$ is the cosmological scale factor (function of the cosmic time t), we have the relation

$$\frac{\dot{a}}{a} = H = -\frac{\dot{z}}{z+1} \, , \tag{2}$$

so that all quantities containing the Hubble parameter H or the red–shift z have to oscillate. Then the oscillation in red–shift can be considered, as first proposed by Tifft (Tifft 1977, Tifft 1987), as a sort of "quantization". On the other hand, the apparent periodicity in the distribution of galaxies implies peaks in the red–shift distribution which lay on concentric spherical shells with periodically spaced radii (someone claimed for a new Tolemaic principle from the point of view of Earth!).

The main quantities affected by oscillations are the following.

1. The number count–red–shift relation of galaxies

$$\frac{dN}{d\Omega dL dz} = n(L, t_0) a_0^2 H^{-1} \mathbf{d}^2 \,, \tag{3}$$

where dN is the number of galaxies in the solid angle $d\Omega$ having red–shift between z and $z + dz$ and luminosity between L and $L + dL$; $n(L, t_0)$ is the number density of galaxies with luminosity L that an observer sees at t_0; a_0 is the actual scale factor and $\mathbf{d}$ is the comoving distance defined as $\mathbf{d} = \int_t^{t_0} \frac{dt}{a}$. If H oscillates, the number of galaxies in a solid angle changes as a function of the red–shift and the luminosity.

2. The two–point correlation function which can be rewritten in term of H as

$$\xi(r) = \left(\frac{\mathbf{d}_s}{r_s}\right)^{-3} \left(\frac{\mathbf{d}}{r}\right)^2 \left(\frac{H_0}{H}\right) - 1 \,, \tag{4}$$

where r is the red–shift distance (Weinberg 1972)

$$r = z H_0^{-1} = \sqrt{\frac{L}{4\pi F}} \,, \tag{5}$$

with L the luminosity of the object and F the measured flux; d_s is the comoving distance of the sample, r_s is the sample size, which can be $r_s \simeq 5h^{-1}$Mpc for the galaxies and $r_s \simeq 25h^{-1}$Mpc for the clusters; H_0 is the actual Hubble constant. However, the oscillations can enhance or depress the galaxy–galaxy correlation function since the distance between the objects apparently increases or decreases.

3. The number count–redshift relation for quasars. Since the oscillations of the Hubble parameter is coherent and global, we expect a similar clustering property also in higher red–shift regions such as $1 \leq z \leq 6$.

In general, given a sample of homologous objects spaced in red–shift, we can, in principle, apply the same argument used for the galaxies, that is we can expect an apparent periodicity in the distributions (*e.g.* Lyman α–clouds, radio galaxies, and so on).

Besides we can ask for some fundamental physics explaination of such a phenomenology as the above mentioned dynamics of a scalar field. For example, some authors are

recently supposing a deep link between quantization and gravitation. Some of them support the point of view that gravity could give rise to a sort of "stochastic" quantization which relates the cosmic scales of interest to the Planck constant (Calogero 1997). Others search for extensions of Einstein's relativity able to include, for examples, fractal structures (Nottale 1997). A more conservative point of view could be considering cosmic structures as "remnants" of primordial quantum processes and trying to connect early "quantum" dynamics with today–observed classical macroscopic dynamics.

This is, in some sense, the usual approach of most of quantum gravity theories which consider either the whole universe as a quantum system (supposing, for example, several co–existing, noninteracting universes (Everett 1957, De Witt 1967, Vilenkin 1982) or the universe as a classical background where primordial quantum processes have given rise to the actual macroscopic structures (this is the approach of quantum field theory on curved spacetimes (Birrell & Davies 1982).

In relation to the first point of view, it has been shown that the Wheeler–DeWitt equation $\mathcal{H}\psi = 0$ for the wave function of the universe ψ can be written, in the Friedman–Robertson–Walker (FRW) spacetime, as a Schrödinger equation $i\xi\dfrac{\partial\psi}{\partial\alpha} = \tilde{\mathcal{H}}\psi$, where $\xi = \dfrac{d\alpha}{dt}$ (Pollock 1997). In this sense (*i.e.* in the mini–superspace approximation), quantum gravity can be treated as ordinary quantum mechanics. This approximation could imply experimental verifications of the Wheeler–DeWitt equation (Pollock 1997, Colella *et al.*1975) and give further constraints on big–bang nucleosynthesis (Llorente & Peréz-Mercader 1995).

In this paper, following Rosen 1993, we want to reduce, at least formally, the cosmological Einstein-Friedman equations of general relativity to a quantum mechanical system. If this issue holds, the Friedman equations can be recast as a Schrödinger equation and the cosmological solutions can be read as eigensolutions of such a "cosmological" Schrödinger equation. In this quite simple scheme, the bound states (with negative energy) are automatically oscillating functions and the probability amplitudes can fit the BEKS oscillatory distribution with an opportune choice of the parameters: that is, it is nothing else but a particular eigensolution. This means that the observed oscillations are just the remnant of a primordial quantum state enlarged by the cosmological expansion (which has been huge if an early inflationary phase is supposed). An important point has to be stressed: here, we are not considering the above mentioned minisuperspace approach in which one estimates the probability that a particular "classical" universe comes out from quantum boundary conditions (Everett 1957, De Witt 1967, Vilenkin 1982) : we are just recasting the cosmological dynamical system as a Schrödinger quantum–like system where bounded solutions could give rise to the observed large scale structure. The probability has to be connected to the above correlation function.

The paper is organized as follows. In Sec.2, following the approach by Rosen (Rosen 1993), we construct our cosmological Schrödinger equation. Sec.3 is devoted to solve such an equation stressing the solutions characterized by oscillatory behaviours. Discussion of the results and conclusions are drawn in Sec.4.

2   Cosmological equations as a Schrödinger equation

The cosmological evolution equations come from the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}\,, \tag{6}$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the stress–energy tensor, and Λ is the cosmological constant which is taken into consideration in order to generalize the discussion. Such equations can be derived from the Hilbert–Einstein action

$$\mathcal{A} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R + 2\Lambda + \mathcal{L}_m]\,, \tag{7}$$

where $\mathcal{L}_m$ is the Lagrangian density relative to the matter fields. As a simple hypothesis, we consider standard perfect fluid matter so that, by varying $\mathcal{L}_m$ with respect to $g_{\mu\nu}$, we get

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}\,, \tag{8}$$

where ρ and p are the energy–matter density and the pressure respectively. The dynamical problem is completely set when matter evolution equations are given; they are the contracted Bianchi identities

$$T^\mu_{\nu;\mu} = 0\,. \tag{9}$$

A further equation has to be imposed in order to assign the thermodynamical state of matter. It, usually, is

$$p = (\gamma - 1)c^2\rho\,, \tag{10}$$

where γ is a constant ($1 \leq \gamma \leq 2$ for standard perfect fluid matter).

The Einstein–Friedman cosmological equations are easily recovered as soon as the Friedman–Robertson–Walker metric

$$ds^2 = c^2dt^2 - a^2(t)\left\{\frac{dr^2}{1 - kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right\}\,, \tag{11}$$

is imposed. Here $a(t)$ is the scalar factor of the universe, $k = 0, \pm 1$ is the curvature constant.

Then, from Eqs.(6) and (9), we obtain the system

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3p) + \frac{\Lambda c^2}{3}\,, \tag{12}$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3c^2}\rho + \frac{\Lambda c^2}{3}\,, \tag{13}$$

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0\,, \tag{14}$$

which has to be completed by Eq.(10).

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Let us now rewrite the $(0,0)$–Einstein equation (13) as

$$\frac{1}{2}m\dot{a}^2 - \frac{m}{6}\left(\frac{8\pi G}{c^2}\rho + \Lambda c^2\right)a^2 = -\frac{1}{2}mkc^2\,, \tag{15}$$

where m is a mass which will be specified below.

We can formally write, following Rosen (Rosen 1993)

$$T + V = E\,, \tag{16}$$

where

$$T = \frac{1}{2}m\dot{a}^2\,, \tag{17}$$

$$V(a) = -\frac{4\pi mG}{3c^2}\rho a^2 - \frac{m\Lambda c^2 a^2}{6}\,, \tag{18}$$

and

$$E = -\frac{1}{2}mkc^2\,. \tag{19}$$

T has the role of the kinetic energy, $V(a)$ of the potential energy, and E of the total energy of the system.

Since Eq.(13) is the first integral of Eq.(12), the equation of motion of a "particle" of mass m is recovered, being

$$m\ddot{a} = -\frac{dV}{da}\,, \tag{20}$$

and using Eq.(14).

By this formalism, we can define a "particle" momentum

$$\Pi = m\dot{a}\,, \tag{21}$$

and the Hamiltonian

$$\mathcal{H} = \frac{\Pi^2}{2m} + V(a)\,. \tag{22}$$

Using the simple scheme of first quantization, we can substitute

$$\Pi \to -i\hbar\frac{\partial}{\partial a}\,, \tag{23}$$

and then derive the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial a^2} + V(a)\Psi\,, \tag{24}$$

where the wave function is

$$\Psi = \Psi(a,t)\,. \tag{25}$$

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In our case, m can be interpreted as the mass of a galaxy and $|\Psi|^2$ as the probability to find such a galaxy at a given $a(t)$. Immediately, we can pass from $a(t)$ to the observable red–shift z by the relation

$$\frac{a_0}{a} = 1 + z\,, \tag{26}$$

where a_0 is the actual scale factor. In this sense, $\Psi = \Psi(z,t)$ is the formal probability amplitude to find a given object of mass m at a given red–shift z, at time t.

Following the ordinary quantum mechanics, a stationary state of energy E is

$$\Psi(a,t) = \psi(a)e^{-iEt/\hbar}\,, \tag{27}$$

and the Schrödinger stationary equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{da^2} + V(a)\psi = E\psi\,, \tag{28}$$

or, in terms of z,

$$-\frac{\hbar^2}{2m}\left[\frac{(z+1)^4}{a_0^2}\frac{d^2\psi}{dz^2} + \frac{2(z+1)^3}{a_0^2}\frac{d\psi}{dz}\right] + V(z)\psi = E\psi\,, \tag{29}$$

being

$$V(z) = -\left(\frac{4\pi mG}{3c^2}\right)\frac{\rho(z)a_0^2}{(1+z)^2} - \frac{m\Lambda c^2}{6(1+z)^2}\,. \tag{30}$$

We shall continue our analysis by using the scale factor $a(t)$, since the results can be easily translated in z.

To conclude this section, we have to stress that our model differs from that in Rosen (Rosen 1993), since there m was the mass of the whole universe, while we are using an effective "particle", which, from a cosmological point of view, can be a galaxy. Furthermore, we propose a different probability interpretation for Ψ.

3   The eigensolutions

From Eqs.(10) and (14), we obtain

$$\rho = Aa^{-3\gamma}\,, \tag{31}$$

where A is a positive integration constant which gives the density of standard matter at a given epoch a_0. The thermodynamical state of matter is assigned by giving γ. We have, for example, $\gamma = 1$ for dust–like matter and $\gamma = 4/3$ for radiation.

The Schrödinger stationary equation (28) can be written in the form

$$\psi'' + \left[Ba^{(2-3\gamma)} + Ca^2 + \mathcal{E}\right]\psi = 0\,, \tag{32}$$

where

$$B = \frac{8\pi G m^2}{3\hbar^2} A \,, \quad C = \frac{m^2 c^2 \Lambda}{3\hbar^2} \,, \quad \mathcal{E} = \frac{2mE}{\hbar^2} = -\frac{mc^2 k}{2} \,, \tag{33}$$

and the prime indicates the derivative with respect to a. Such a parametric differential equation can be exactly solved in several cases giving rise to oscillatory behaviours. In any case, being nothing else but the one–dimensional problem of ordinary quantum mechanics, we can discuss its general features, in particular when oscillatory solutions are present.

3.1 General considerations

Let us rewrite Eq.(32) as

$$\psi'' + [\mathcal{E} - U(a)] \, \psi = 0 \,, \tag{34}$$

where

$$U(a) = - \left[Ba^{(2-3\gamma)} + Ca^2 \right] \,. \tag{35}$$

By comparing the potential $U(a)$ and the "energy" $\mathcal{E}$, we can discuss the qualitative behaviour of solutions. However, the form of potential strictly depends on the parameters B, γ, and C (alternatively on A, γ, and Λ) so that, due to the meaning which we are going to give to ψ, the clustering and correlation properties are functions, as it must be, of the kind and of the content of the fluids involved into the dynamics. Furthermore, the energy is "fixed" by the given FRW–spatial model.

If we require that we cannot find galaxies at infinity, the function $\psi(a)$ is normalized. As a trivial result, the function $\psi(a) = 0$ is a solution of the problem. To avoid that this is the *only* solution, we must ask for nontrivial normalized solutions which will set up a discrete spectrum.

By fixing the energy (that is by choosing the FRW model), we can divide the a–axis in a certain number of intervals using the points where $\mathcal{E} - U(a) = 0$. We assume, for simplicity, that such points are a finite number. Then, the extreme intervals have an unbounded extension, and, between them, we have a finite number of bounded intervals. We call type–I intervals those where $\mathcal{E} < U(a)$ so that $\psi''/\psi > 0$, and type–II intervals those where $\mathcal{E} > U(a)$, so that $\psi''/\psi < 0$. In type–I intervals, the function ψ never changes its sign, in type–II intervals, it is, in general, oscillating and can change it sign. In this case we could have solutions showing clustering and anti–clustering properties for galaxies. We have bending points if $\mathcal{E} = U(a)$: in these cases, the solution transits from type–I to type–II intervals and *viceversa*. The function is normalizable if the extreme intervals are of type I. In fact, if we are considering type–II intervals, the "particle" can escape towards infinity, that is it can stay at extremely great distance from a given point. This fact is physically in contrast with any clustering or correlation property. In other words, the discrete spectrum is inside the energy values which do not allow the particle to escape towards infinity. For this reason, the energy must be less than $U(a)$. However, we have to stress that the situation $t \to -\infty$ means that we are considering singularity–free models but $a(t)$ has to remain nonnegative.

If we are in some regime where $|\mathcal{E}| \gg U(a)$, with $\mathcal{E}$ positive defined (*i.e.* $k = -1$), Eq.(34) has stationary wave solutions of the form

$$\psi(a) = a_1 e^{i\sqrt{\mathcal{E}}a} + a_2 e^{-i\sqrt{\mathcal{E}}a} \,, \tag{36}$$

if the energy is negative, the physical solutions are decreasing exponential functions. Such functions are not always normalizable but it is easy to see that oscillatory behaviours can be recovered.

3.2 Special cases

From Eq.(34), by choosing suitable values of the parameters B, γ, C, and $\mathcal{E}$, we can recover all the standard cases of ordinary quantum mechanichs (potential well, harmonic oscillator, Coulomb–like potential, and so on). As it is well known (Messiah 1961), many of them exhibit oscillatory eigensolutions. Considering our point of view, the interesting values of γ are $\gamma = 4/3$ (radiation), $\gamma = 1$ (dust), $\gamma = 0$ (scalar field).

3.2.1 *The radiation case*

In this case, Eq.(32) becomes of the form

$$\psi'' + \left[\frac{B}{a^2} + Ca^2 + \mathcal{E}\right]\psi = 0 \,. \tag{37}$$

For $\Lambda = 0$ and $k = -1$, it can be written as

$$\psi'' + \left[\beta^2 - \frac{(\nu^2 - 1/4)}{a^2}\right]\psi = 0 \,, \tag{38}$$

with

$$\beta^2 = \mathcal{E} = \frac{mc^2}{2} \,, \quad \nu = \sqrt{\frac{1}{4} - B} \,, \tag{39}$$

which is a Bessel equation. The general solution is (Abramowitz & Stegun 1970)

$$\psi(a) = \sqrt{a} Z_\nu(\beta a) \,. \tag{40}$$

From the theory of Bessel functions, $Z_\nu(x)$ can be given as a combination of the functions $J_\nu(x)$, $Y_\nu(x)$, or $H_\nu^{(1)}(x)$, $H_\nu^{(2)}(x)$. The analysis of asymptotic behaviours results particularly interesting. We have, for $a \to 0$,

$$
\begin{aligned}
J_\nu(a) &\sim \left(\frac{a}{2}\right)^\nu \Gamma(\nu + 1) \,, \\
Y_0(a) &\sim -iH_0^{(1)}(a) \sim iH_0^{(2)}(a) \sim \frac{2}{\pi}\ln a \,, \\
Y_\nu(a) &\sim -iH_\nu^{(1)}(a) \sim iH_\nu^{(2)}(a) \sim -\frac{1}{\pi}\Gamma(\nu)\left(\frac{a}{2}\right)^{-\nu} \,.
\end{aligned} \tag{41}
$$

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In the opposite situation, that is $a \to \infty$, we have oscillating solutions

$$J_\nu(a) \sim \sqrt{\frac{2}{\pi a}} \cos\left(a - \frac{\nu\pi}{2} - \frac{\pi}{4}\right),$$

$$Y_\nu(a) \sim \sqrt{\frac{2}{\pi a}} \sin\left(a - \frac{\nu\pi}{2} - \frac{\pi}{4}\right),$$

$$H_\nu^{(1)}(a) \sim \sqrt{\frac{2}{\pi a}} \exp\left[i\left(a - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\right], \tag{42}$$

$$H_\nu^{(2)}(a) \sim \sqrt{\frac{2}{\pi a}} \exp\left[-i\left(a - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\right]. \tag{43}$$

For $\Lambda = 0$ and $k = +1$, we get

$$\psi(a) = \sqrt{a} Z_\nu\left(i\sqrt{|\mathcal{E}|}a\right), \tag{44}$$

where the relations

$$J_\nu(ia) = e^{i\pi\nu} I_\nu(a),$$

$$Y_\nu(ia) = e^{i(\nu+1)\pi/2} I_\nu(a) - \frac{2}{\pi} e^{-i\pi\nu} K_\nu(a),$$

$$H_\nu^{(1)}(ia) = -i\frac{2}{\pi} e^{-i\pi\nu/2} K_\nu(a), \tag{45}$$

hold.

If $k = 0$ but $\Lambda \neq 0$, we have

$$\psi(a) = \sqrt{a} Z_\nu\left(\frac{\sqrt{C}}{2}a^2\right), \tag{46}$$

where

$$\nu = \frac{\sqrt{1 - 4B}}{4}. \tag{47}$$

3.2.2 The dust case

In the dust–dominated case, we have

$$\psi'' + \left[\frac{B}{a} + Ca^2 + \mathcal{E}\right]\psi = 0, \tag{48}$$

and for $k = \Lambda = 0$, we have solutions of the form

$$\psi(a) = \sqrt{a} Z_\nu(\sqrt{a}), \tag{49}$$

where the above considerations hold. If $C = 0$, we get an hydrogenlike Schrödinger equation (with null angular momentum) and the wavefunction $\psi(a)$ can be written as a combination of Laguerre polynomials (see also Rosen 1993, Messiah 1961).

3.2.3 *The scalar field case*

The scalar field case, is not exactly a standard perfect fluid form of matter but it is of extreme physical interest since it is connected to inflation (Kolb & Turner 1990). However, it can be treated as a standard fluid assuming $\gamma = 0$ in the equation of state Eq.(10) (Guth 1981). From the point of view of dynamics, it has the same role of cosmological constant so that Eq.(32) becomes

$$\psi'' + \left[Ca^2 + \mathcal{E} \right] \psi = 0 \,. \tag{50}$$

We get the same equation by putting $B = 0$. However, the cosmological constant must be redefined. It is clear that, depending on the relative signs and values of C and $\mathcal{E}$, Eq.(50) reduces to the equation of an harmonic oscillator with combinations of Hermite polynomials as eigensolutions or, in general, to the differential equation of parabolic cylinder functions (Abramowitz & Stegun 1970).

4 Discussion and Conclusions

Several theoretical interpretations of BEKS's result have been attempted so far (Busarello *et al.*1994, Morikawa 1991, Capozziello 1996, Budinich & Raczka 1993) but in all of them modifications of Einstein standard theory of gravity have been invoked. Essentially, a further ingredient as a scalar field should have had the role of perturbing the overall Friedmanian dynamics of the universe breaking the homogeneity and isotropy at scales of the order of $\sim 100 h^{-1} \mathrm{Mpc}$. The approach which we have adopted in this paper is, in some sense, absolutely conservative since we are still using Einstein general relativity. The only issue is that the Friedman–Einstein cosmological dynamics could be read in the sense of elementary quantum mechanics: the $(0,0)$–energy equation should give the Schrödinger equation in a simple scheme of first quantization. In order to justify such an assumption, we must suppose a primordial quantum dynamics of matter contained in the universe and the fact that the today observed large scale structure reflects the phenomenology of early epochs. In other words, we can say that the Schrödinger Eq.(32) describes exactly the behaviour of the universe for any value of the cosmic time and the classical Einstein equations (in particular the $(0,0)$) well reflect the evolution of dynamical system when it can be described at a classical level.

Here we have not considered any questions connected neither to quantum field theory nor to quantum gravity. We are simply taking into account the "particle" dynamics of objects which are galaxies. The probability of finding them at a certain epoch (*i.e.* $a(t)$) or a certain red–shift is given by the Schrödinger equation which is nothing else but the quantum counterpart of the classical Einstein first order equation. Furthermore, the eigensolutions of such an equation, having a probabilistic meaning, can be connected to the correlation function which gives the features of the distribution of a certain class of objects (*e.g.* galaxies, clusters or superclusters of galaxies).

As we have shown, several possibilities exist to get oscillatory behaviours: they depend on the kinds and the number of fluids into the dynamics and, essentially, on the

kind of FRW spatial model. If the universe is dust–dominated, with a remnant of cosmological constant, the asymptotic behaviour of a Bessel function could easily reproduce the BEKS observations (in Budinich 1993, Gegenbauer's polynomial are used). A good match is obtained if $\psi(a)$, that is $\psi(z)$, has 8 oscillations with periodicity of $128h^{-1}$Mpc in $2000h^{-1}$Mpc (*i.e.* in a red–shift range from $z = 0$ to $z = 0.5$). The amplitude of such oscillations strictly depends on the cosmological densities of fluids involved (see Busarello *et al.*1994 for a detailed discussion).

In conclusion, we can say that a breaking of homogeneity and isotropy at small scales ($\sim 100h^{-1}$Mpc) with oscillating correlations (and anticorrelations) between galaxies could be easily implemented considering a sort of cosmological Schrödinger equation.

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